4. R. D. Small, V. A. Sernas and R. H. Page, Single beam schlieren interferometer using a Wollaston prism, *Appl. Optics* **11**, 858–862 (1972).
5. W. Merzkirch, Generalized analysis of shearing interferometers as applied for gas dynamic studies, *Appl. Optics* **13**, 409–413 (1974).
6. W. Merzkirch and W. Erdmann, Evaluation of axisymmetric flow patterns with a shearing interferometer, *Appl. Phys.* **2**, 119–122 (1973).
7. E. M. Sparrow and J. L. Gregg, Laminar-free-convection heat transfer from the outer surface of a vertical circular cylinder, *J. Heat Transfer* **78**, 1823–1829 (1956).
8. R. Viskanta, The axisymmetric free-convection heat transfer along a vertical thin cylinder with constant surface temperature, Argonne National Laboratory, Rept. ANL-6660 (1963).
9. A. A. Hayday, D. A. Bowlus and R. A. McGraw, Free convection from a vertical flat plate with step discontinuities in surface temperature, *J. Heat Transfer* **89**, 244–250 (1967).

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ZERO LATENT ROOTS OF THE COEFFICIENT MATRIX IN THE EQUATION OF MULTICHANNEL EXCHANGERS

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NOMENCLATURE

a_{ij},	number of transfer units per unit of coordinate x;
e_i,	column vector of $\mathbf{K}$;
h_{ij},	common perimeter of channels i and j;
k_{ij},	overall surface conductance for heat transfer between channels i and j;
n,	number of channels;
t_i,	temperature of fluid in channel i;
x,	space coordinate along the channel;
$\boldsymbol{x}$,	column vector;
$\mathbf{A}$,	coefficient matrix;
$\mathbf{B}$,	diagonal matrix;
$\mathbf{C}$,	$n \times n$ matrix;
$\mathbf{S}$,	$n \times n$ symmetric matrix;
$\boldsymbol{t}$,	temperature column vector, $\boldsymbol{t} = [t_1, t_2, \ldots, t_n]^T$;
W,	fluid heat capacity rate.

CONSIDER the equation

$$\frac{d\boldsymbol{t}}{dx} = \mathbf{A}\boldsymbol{t} \tag{1}$$

where $\boldsymbol{t}$ is the column matrix of temperature and $\mathbf{A}$ is a square, non-block diagonal matrix of order n and of rank equal to $n-1$ [1] defined by

$$\mathbf{A} = \begin{bmatrix} a_{11} a_{12} \ldots a_{1n} \\ \ldots\ldots\ldots\ldots \\ a_{n1} a_{n2} \ldots a_{nn} \end{bmatrix} \tag{2}$$

$$a_{ij} = \frac{k_{ij}h_{ij}}{W_i} \qquad (i \neq j,\ k_{ij}h_{ij} = k_{ji}h_{ji})$$

$$a_{ii} = -\frac{1}{W_i}\sum_{j=1}^{n} k_{ij}h_{ij} \qquad (k_{ii}h_{ii} = 0).$$

The solution $\boldsymbol{t}$ depends on the features of the coefficient matrix $\mathbf{A}$.

The properties of $\mathbf{A}$ have been discussed by other authors. For example in [2, 3] it is shown that all the latent roots are real and in [1] it is proved that the necessary and sufficient condition for $\mathbf{A}$ to have at least two zero latent roots is

$$\sum_{i=1}^{n} W_i = 0.$$

It results from the foregoing, that to characterize the spectrum of $\mathbf{A}$ one problem still remains, namely the maximum multiplicity of zero latent roots.

Two lemmas will be helpful in answering this question.

Lemma 1

If $\mathbf{S} = [s_{ij}]$ is a symmetric matrix of order n, $\boldsymbol{x} = (x_1, \ldots, x_n)$ any column vector and $\boldsymbol{y}$ a proper vector of $\mathbf{S}$ corresponding to zero latent root, then the scalar product

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{y}) = 0.$$

The proof depends on the property of a symmetric matrix that

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{x}, \mathbf{S}\boldsymbol{y}) = (\boldsymbol{x}, \mathbf{0}) = 0.$$

Lemma 2

If $\boldsymbol{x}$ and $\mathbf{S}$ are given as in lemma 1 and $\mathbf{S}$ is a semidefinite matrix (positive or negative), then only a proper vector, say $\boldsymbol{x}$, of $\mathbf{S}$ appropriate to a zero latent root satisfying

$$\mathbf{S}\boldsymbol{x} = \mathbf{0},$$

may be a non-trivial solution of the equation

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{x}) = (\boldsymbol{x}, \mathbf{S}\boldsymbol{x}) = 0.$$

Proof

Let $\mathbf{S}$ be negative semidefinite. Then it follows that

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{x}) \leqslant 0;$$

furthermore

$$(\mathbf{S}\boldsymbol{x}, \boldsymbol{x}) = \sum_{i,j=1}^{n} s_{ij}x_ix_j$$

is a continuous function of the n variables $x_1, x_2, \ldots, x_n$ possessing continuous first and second partial derivatives.

Consequently the solution of equation

$$(\mathbf{S}x, x) = 0$$

is equivalent to determining the maximum of $(\mathbf{S}x, x)$; this leads to

$$\frac{\partial(\mathbf{S}x, x)}{\partial x_1} = \frac{\partial(\mathbf{S}x, x)}{\partial x_2} = \ldots = \frac{\partial(\mathbf{S}x, x)}{\partial x_n} = 0$$

and gives

$$s_{k1}x_1 + s_{k2}x_2 + \ldots + s_{kn}x_n = 0 \qquad (k = 1, 2, \ldots, n)$$

which may be expressed in the form

$$\mathbf{S}x = \mathbf{0}.$$

If $\mathbf{S}$ is a positive semidefinite matrix the proof is similar to the above.

Corollary

If $\mathbf{S}$ fulfils assumptions of lemma 2 and $\mathbf{S}x \neq \mathbf{0}$, then

$$(\mathbf{S}x, x) = (x, \mathbf{S}x) \neq 0.$$

Theorem

The multiplicity of the zero latent root of $\mathbf{A}$ does not exceed two.

For the proof let us remark [1, 2] that $\mathbf{A}$ can be represented by the product.

$$\mathbf{A} = \mathbf{B}^{-1}\mathbf{C}$$

where

$$\mathbf{B} = \begin{bmatrix} W_1 & & & & 0 \\ & W_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ 0 & & & & W_n \end{bmatrix}$$

and

$$\mathbf{C} = [c_{ij}] \qquad 1 \leqslant i, j \leqslant n$$

with

$$c_{ij} = k_{ij}h_{ij} = k_{ji}h_{ji} = c_{ji} \qquad (i \neq j),$$

$$c_{ii} = -\sum_{j=1}^{n} k_{ij}h_{ij} \qquad (k_{ii}h_{ii} = 0).$$

Further $\mathbf{A}$ can be reduced to its Jordan canonical form $\mathbf{J}$, where

$$\mathbf{A} = \mathbf{KJK}^{-1} \tag{3}$$

or

$$\mathbf{AK} = \mathbf{KJ}.$$

As $\mathbf{A}$ is of rank $n-1$, then in the structure of $\mathbf{J}$ only one Jordan submatrix can be distinguished. The form of $\mathbf{J}$ is

$$\mathbf{J} = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & . & . & s \end{array} \\ \left[\begin{array}{ccccc|c} 0 & 1 & & & 0 & \\ & 0 & 1 & & & 0 \\ & & 0 & \cdot & & \\ & & & 0 & 1 & \\ & & & & 0 & \\ \hline & 0 & & & & \mathbf{J}' \end{array}\right] \end{array} \begin{array}{c} 1 \\ 2 \\ . \\ . \\ . \\ s \end{array} \tag{4}$$

The structure of $\mathbf{J}'$ will not be important for further considerations.

The order of the Jordan submatrix s is equal to multiplicity of zero latent roots.

The transforming matrix $\mathbf{K}$ is built of n vectors $e_1, \ldots, e_s, \ldots, e_n$ which are forming the set of linearly independent columns. Hence.

$$\mathbf{K} = [e_1, e_2, \ldots, e_s, \ldots, e_n].$$

Note that only e_1 is the eigenvector corresponding to zero latent root.

Regarding the above notation (3) may be written in the form

$$\mathbf{B}^{-1}\mathbf{C}[e_1, e_2, \ldots, e_s, \ldots, e_n] = [e_1, \ldots, e_s, \ldots, e_n]\mathbf{J}. \tag{5}$$

From (4) and (5) it follows that

$$\begin{aligned} \mathbf{B}^{-1}\mathbf{C}e_1 &= \mathbf{0} \quad \nRightarrow \mathbf{C}e_1 = \mathbf{0} \\ \mathbf{B}^{-1}\mathbf{C}e_2 &= e_1 \quad \nRightarrow \mathbf{C}e_2 = \mathbf{B}e_1 \\ \mathbf{B}^{-1}\mathbf{C}e_3 &= e_2 \quad \nRightarrow \mathbf{C}e_3 = \mathbf{B}e_2 \\ &\ldots \\ \mathbf{B}^{-1}\mathbf{C}e_s &= e_{s-1} \nRightarrow \mathbf{C}e_s = \mathbf{B}e_{s-1}. \end{aligned} \tag{6}$$

For the proof it is sufficient to show that the order s of the Jordan submatrix (4) does not exceed two.

(a) It is apparent that the Jordan submatrix of order $s = 1$ always exists.

(b) For the existence of the Jordan submatrix of order $s = 2$, two first equations of (6) have to be satisfied. Vector e_1 can be calculated from the first of these and as all the cofactors of $\mathbf{C}$ are equal [1], then it has the form

$$e_1 = (1, 1, \ldots, 1).$$

Vector e_2 has to satisfy the second equation (6) and also, according to lemma 1, the relation

$$(\mathbf{C}e_2, e_1) = 0$$

or

$$(\mathbf{B}e_1, e_1) = 0$$

hence, when the structure of e_1 and $\mathbf{B}$ is regarded, well known relation [1] is obtained.

$$W_1 + W_2 + \ldots + W_n = 0.$$

(c) For $s \geqslant 3$ at least three first equations of (6) have to be satisfied, then

$$\begin{aligned} \mathbf{C}e_1 &= \mathbf{0} \\ \mathbf{C}e_2 &= \mathbf{B}e_1 \\ \mathbf{C}e_3 &= \mathbf{B}e_2 \end{aligned} \tag{6a}$$

but it results also from lemma 1 that

$$(\mathbf{C}e_3, e_1) = 0$$

and, as $\mathbf{B}$ is a symmetrix matrix, we obtain further

$$(\mathbf{B}e_2, e_1) = (e_2, \mathbf{B}e_1) = 0.$$

The substitution of the second equation of (6a) into the one above gives

$$(e_2, \mathbf{C}e_2) = 0.$$

As $\mathbf{C}e_2 \neq \mathbf{0}$, the assumption of $s \geqslant 3$ leads to the contradiction with the corollary of lemma 2, completing the proof of the theorem.

REFERENCES

1. T. Zaleski and A. B. Jarzebski, Remarks on some properties of the equation of heat transfer in multichannel exchangers, *Int. J. Heat Mass Transfer* **16**, 1527–1530 (1973).
2. A. Settari, Remarks about "General solution of the equations of multichannel heat exchangers", *Int. J. Heat Mass Transfer* **15**, 555–557 (1972).
3. J. C. Chato, R. J. Laverman and J. M. Shah, Analysis of parallel flow, multistream heat exchangers, *Int. J. Heat Mass Transfer* **14**, 1691–1703 (1971).

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ANALYSIS OF LOWE'S MEASUREMENTS OF EFFECTS OF VIBRATION ON HEAT TRANSFER

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NOMENCLATURE

a,	amplitude of oscillation;
d,	cylinder diameter;
g,	acceleration due to gravity;
H,	heat-transfer factor (defined in text);
h,	heat-transfer coefficient;
k,	thermal conductivity;
R,	cylinder radius;
ΔT,	difference in temperature between cylinder surface and distant air;
β,	thermal coefficient of expansion of air;
ε,	$4Re_s/Gr^{1/2}$;
λ,	ratio of thicknesses, natural convection boundary layer to harmonic oscillation boundary layers;
ν,	kinematic viscosity;
ω,	circular frequency of imposed harmonic oscillation;
Gr,	Grashof number, $g\beta\Delta TR^3/\nu^2$;
Nu,	Nusselt number, hd/k;
Re_s,	streaming Reynolds number, $a^2\omega/\nu$.

Recent analysis of the effects of vibrations on heat transfer occurring otherwise by pure natural convection have shown a significant role should be played by a boundary-layer thickness parameter λ [1]. Experimental data obtained by Lowe [2] happen to fall in a range of λ where effects of λ should be strong, and allow some comparisons which are described here.

Effects of vibration and sound fields on heat and mass transfer have often been measured for a circular cylinder with the oscillations transverse to its axis. There are several different classes of flow situations which can dominate the transfer process, and no single correlation can reasonably be used to fit all the data [3]. Even in the absence of natural convection effects, data may fall into at least three distinct characteristic solutions in one of which convection is dominated by outer streaming at large streaming Reynolds numbers [4]. Analysis of combined natural convection and horizontal or vertical oscillations at a heated horizontal cylinder [1] for large Grashof and large streaming Reynolds numbers, predicts local changes in boundary-layer thickness and heat transfer; these changes correspond to the directions of local changes observed in experiments at large Grashof and small streaming Reynolds numbers [5].

The analysis [1] draws attention to the additional characteristic parameter $\lambda = R/Gr^{1/4}(2\nu/\omega)^{1/2}$, the ratio of the natural convection boundary-layer thickness to the oscillating boundary-layer thickness. At any finite value of $\varepsilon = 4\,Re_s/Gr^{1/2} = 4(a^2\omega/\nu)/Gr^{1/2}$, the change in heat-transfer increases with λ, rapidly at small values of $\lambda(<10$, say), and approaches a finite limit as $\lambda \to \infty$.

Experimental data at large values of streaming Reynolds number $Re_s = a^2\omega/\nu$ are most easily obtained when ω is small because a can be quite large then. When ω is small, however, the oscillation boundary-layer thickness $(2\nu/\omega)^{1/2}$ is relatively large and values of λ may typically fall in the range 2 to 10, which complicates correlation of data because of the relatively strong effect of λ in this range. At present there does not seem to be local data available at large Re_s to compare with the predictions of the analysis that horizontal oscillations increase the heat-transfer rate at the bottom of the cylinder, and that vertical oscillations decrease the heat transfer, as ε is increased from zero to moderate values. (As ε becomes large, one must expect effects of natural convection to become unimportant.) However, the unpublished thesis of Lowe [2] contains data for overall heat transfer at moderate Grashof numbers (about 3×10^3) and with streaming Reynolds numbers up to 800 ("large"), and it is worthwhile examining these data in the light of the analyses.

The following points can be established about Lowe's data:

(1) The experimental results merge with the two pertinent asymptotes. Figure 1, in which $(Nu/Gr^{1/4})\{1+0{\cdot}94(a/d)\}$ is plotted as a function of ε, shows how the data approach the asymptotic cases of $\varepsilon \to 0$ (pure natural convection) and $\varepsilon \to \infty$ (acoustic streaming dominant) [4]. The value of $Nu/Gr^{1/4}$ expected as $\varepsilon \to 0$ is somewhat larger than that for large Gr (i.e. for the boundary-layer solution as $Gr \to \infty$) because of boundary-layer curvature effects [6].

(2) The rise of *overall* heat transfer as ε increases from zero is slower than the change of local heat transfer predicted by analysis [1] for the same range of λ. A similar observation has been found in other experiments with much higher values of λ [5, 7], where it was found that there are simultaneous local changes of opposing sign and similar magnitude at different locations around the cylinder.

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